

Exercise 17

Differentiate the function.

$$T(z) = 2^z \log_2 z$$

Solution

Rewrite the function first.

$$T(z) = e^{\ln 2^z} \log_2 z = e^{z \ln 2} \log_2 z$$

Then take the derivative.

$$\begin{aligned} T'(z) &= \frac{d}{dz} \left(e^{z \ln 2} \log_2 z \right) \\ &= \left[\frac{d}{dz} (e^{z \ln 2}) \right] \log_2 z + e^{z \ln 2} \left[\frac{d}{dz} (\log_2 z) \right] \\ &= \left[(e^{z \ln 2}) \cdot \frac{d}{dz} (z \ln 2) \right] \log_2 z + e^{z \ln 2} \left(\frac{1}{z \ln 2} \right) \\ &= (e^{z \ln 2} \cdot \ln 2) \log_2 z + \frac{e^{z \ln 2}}{z \ln 2} \\ &= 2^z (\ln 2) \log_2 z + \frac{2^z}{z \ln 2} \\ &= 2^z (\ln 2) \frac{\ln z}{\ln 2} + \frac{2^z}{z \ln 2} \\ &= 2^z \ln z + \frac{2^z}{z \ln 2} \end{aligned}$$